**Special Relativity: Uniform Acceleration in One Dimension and the Relativistic ‘SUVAT’ Equations**

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**Abstract**

An attempt is made to construct a set of relativistic ‘SUVAT’ equations. The equations allow for the calculation of the time dilation experienced by a uniformly accelerating body relative to an observer in an inertial reference frame, as well as the difference between the constant proper acceleration experienced in the accelerating reference frame and the observed acceleration.

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**1. Introduction – Non-Relativistic ‘SUVAT’ Equations**

Consider a particle moving in one dimension with constant acceleration .

The velocity of the particle is therefore simply

where is the initial velocity of the particle. Integrating once more gives us the displacement of the particle at the time *t*:

where is the initial displacement of the particle.

If is taken to be 0, we can find the non-relativistic ‘SUVAT’ equations:

|  |  |  |
| --- | --- | --- |
|  |  | (1.1) |
|  |  | (1.2) |
|  |  | (1.3) |
|  |  | (1.4) |
|  |  | (1.5) |

Our task is to find the relativistic version of the set of equations above. In order to do this, however, we will have to be careful when defining what we mean by a constant acceleration.

**2. Deriving the Relativistic ‘SUVAT’ Equations**

In order to find the correct relativistic description of a constant acceleration, we will start with the Lorentz transformations of the position and time coordinates of a body:

where is the speed of light, is the relative velocity of the new reference frame and is the Lorentz factor:

We can then find the relativistic transformation of velocity:

Now we can find the relativistic transformation of acceleration:

Note that, unlike in non-relativistic kinematics, the acceleration of a body is not the same in all inertial reference frames. In order to find the expression for the acceleration in the comoving frame of the accelerating body, known as the proper acceleration , we simply set to be equal to :

To derive the relativistic ‘SUVAT’ equations, we set to be constant – this what is meant by a constant acceleration when dealing with relativistic kinematics.

We can now start finding the equations. The first step is to rewrite the coordinate acceleration as the rate of change of the velocity:

Rearranging and integrating, we find the first equation:

This is the relativistic form of equation (1.1). Rearranging once more, we find the velocity, ***v***, at the time *t*:

This is the first of the relativistic ‘SUVAT’ equations. Taking ***u***, or the initial velocity, to be 0, the constant of integration is removed. Integrating again gives us the position, ***s***, at the time *t*:

Since ***x0***, or the initial position, is assumed to be equal to 0, the constant of integration is equal to *c*2*/****α*** so that the position of the particle at time *t*=0 is equal to 0. A more useful form of the equation can be found by rearranging the equation to make *t* the subject:

One can now also find the proper time, τ, or the time elapsed in the accelerating frame of reference. Making use of another equation involving the Lorentz factor

one can rewrite the equation relating ***α*** and ***a***:

Integrating this equation and then rearranging the result yields

where the constant of integration is again ignored so that *τ*=0 when ***v***=0.

At this stage, all of the fundamental ‘SτVAT’ (pronounced *stow-vat*) equations have been found. Thus it has been found that it is possible, to some extent, that the relativistic variety of the Newtonian ‘SUVAT’ equations can be derived. This set of equations is most useful when attempting to find the time dilation that occurs, or the difference between the coordinate time, *t*, and the proper time, *τ*, when a journey of some distance, ***s***, is made with constant acceleration in the accelerating frame, ***α***. The observed acceleration, ***a***, can be found after finding the final velocity, ***v***, and using the equation relating ***a*** and ***α***.

For example, a journey to the Andromeda galaxy, 2.4 \* 1022 meters away, starting from rest relative to an observer on Earth, where the constant acceleration in the accelerating frame, ***α***, is equal to *g*, the acceleration due to Earth’s gravity, would appear to take 8.0 \* 1013 seconds, or just over 2.5 million years, to complete to the observer, while the time experienced by the traveller would only be 4.7 \* 108 seconds, or fifteen years. The final acceleration observed back on Earth would be only 5.4 \* 10-19 ms-2, as the final velocity of the traveller would be very close to the speed of light.

Rearranging and mixing these equations may lead to forming a larger set of ‘SτVAT’ equations, but the four derived in this paper are the ones which could be considered the most useful.